KSME International Journal, Vol. 13, No. 12, pp. 905~911, 1999

A New Crack Growth Model for Life Prediction Under Random Loading

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(Received February 13, 1999)

The load interaction effect in variable amplitude fatigue test is a very important issue for correctly predicting fatigue life. Some prediction methods for retardation are reviewed and the problems discussed. The so-called "under-load" effect is also of importance for a prediction model to work properly under random load spectrum. A new model that is simple in form but combines overload plastic zone and residual stress considerations together with Elber's closure concept is proposed to fully take account of the load-interaction effects including both over-load and under-load effects. Applying this new model to complex load sequence is explored here. Simulations of tests show the improvement of the new model over other models. The best prediction (mostly closely resembling test curve) is given by the newly proposed Chen-Lee model.

Key Words : Fatigue, Fatigue Crack Growth, Fatigue Life Prediction, Retardation, Load Interaction, Crack Closure

Nomenclature

a	: (half) Crack length
C	: Coefficient in crack growth rate formula
da/dN	: Crack growth rate
K	: Stress intensity factor (SIF)
K_{ab}	: Boundary SIF required to offset overload plasticity
m, n, w	: Exponents in crack growth rate formula
N	: Number of fatigue cycles
r_{pi}	: Current plastic zone size
R	: Stress ratio
U	: Crack closure factor
γ	: Plane stress constraint factor
γ_C	: A material constant
λ	: Distance from current crack tip to overload plastic zone boundary
η	: The stress relaxation coefficient in HeQZ model
σ	: Stress
σ_y	: Yield stress
Δ	: Varying range of a quantity
ov(OL), eff, res, th, max, min	: Subscripts used for overload, effective, residual, threshold, maximum and minimum of a quantity

1. Introduction

The load interaction effect in variable amplitude fatigue test is a very important issue for correctly predicting fatigue life (Kim et al, 1996 ; Lee, 1998 ; Lee et al ; 1998). The interaction effect includes both overload retardation and under-load acceleration phenomena. Explanations of retardation phenomenon were partly given on the concepts of crack tip plastic zone (Wheeler, 1972), residual stresses (Chang et al, 1981, He, 1981) in the plastic zone, and the crack closure (Elber, 1970), but a fully understanding of load interaction effect still needs much research efforts.

Some prediction methods that can simulate the retardation effect were proposed in literatures (Chang et al, 1981 ; Elber, 1970 ; He, 1981 ; Wheeler, 1972). We are examining some of them

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in light of test results, discussing their merits and problems in the next section. A new model that is simple in form but combines overload plastic zone and residual stress consideration together with Elber's closure concept is proposed in the present paper to take account of the load-interaction effects. Applying this new model to complex load sequence is explored here. Simulations of tests show the improvement of the new model over other models.

2. Existing Models and Their Problems

Wheeler model (Wheeler, 1972) is probably the most widely used retardation model. It is based purely on plastic zone size consideration as shown in Fig. 1, where r_{pi} is the current plastic zone size, λ is the residual retardation zone determined by overload plastic zone boundary.

The retarded rate is obtained by simply multiplying the original (no effect of overload retardation) rate by a retardation factor of

$$Cd = (r_{pi}/\lambda)^{w} \quad (1)$$

i.e. $$da/dN(ret.) = (r_{pi}/\lambda)^{w} \times da/dN(no\ ret.) \quad (2)$$

where w is termed as Wheeler index and is an adjustable fitting parameter.

Wheeler's retardation model is implemented in our own fatigue crack growth (FCG) life prediction program and the a~N relations of Wheeler-Paris simulation for tests with different overload ratios can be found in (Chen et al, 1998). It was shown that the index w is not a material constant but also a function of overload ratio ($OLR = P_{ov}/P_{\max}$), where P_{ov} is overload and $P_{\max}$ is maximum load.

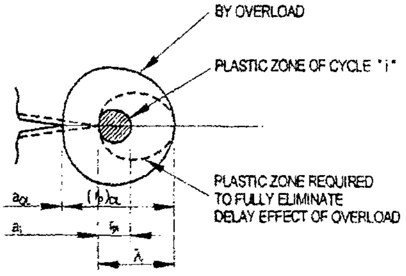


Fig. 1 Plastic zone size in Wheeler model

The Original Willenborg model (Chang et al, 1981) combines the plastic zone size and residual stress with the effective stress intensity factor (SIF) concept. The residual stress within plastic zone caused by the overload must be overcome by the applied SIF, so the effective SIF and R ratio are changed. Crack growth rates that are retarded due to the overload are corrected by substituting effective stress ratio, R_{eff} and effective stress intensity range, ΔK_{eff} into Forman's equation and no additional parameter is needed (Anderson, 1995).

In calculating effective SIF, the λ parameter appears in the model to compute a 'boundary SIF K_{ap}' as

$$K_{ap} = \sigma_y \sqrt{\gamma \pi (a_{ov} + r_{ov} - a)} = \sigma_y \sqrt{\gamma \pi \lambda} \quad (3)$$

The residual SIF and effective SIF are then

$$K_{res} = K_{\max} - K_{ap} \quad (4)$$

$$K_{eff} = K - K_{res} \quad (5)$$

The generalized Willenborg Model (Chang et al, 1981) further modified effective SIF to become

$$K_{eff} = K - \Phi K_{res} \quad (6)$$

where

$$\Phi = \frac{1 - K_{\max,th}/K_{\max}}{\gamma_C - 1} \quad (7)$$

$K_{\max,th}$ and γ_C are the threshold stress intensity factor associated with zero crack growth and a material property, respectively. γ_C may be considered as the shut off overload ratio that can cause complete retardation when the crack ceases to grow ($da/dN = 0$).

Results of applying the Willenborg series models including the original, the generalized and the Willenborg/Chang models to analyze overload tests can also be found in some references (Chang et al, 1981, Chen et al, 1998).

HeQingZhi's model (He, 1981) utilized the residual compress stress concept in plastic zone and also took the stress relaxation into account. It is a model which bridges Willenborg model and Elber's closure concept (Elber, 1970) by using the U factor to calculate effective SIF. The modified effective SIF range is similar to Elber's formula as:

$$\Delta K_{eff} = U \Delta K \tag{8}$$

where

$$U = 1 + (1-\eta)/(1-R) - (1-\eta) K_{ap}/\Delta K \tag{9}$$

In Eq. (9), K_{ap} is the same boundary SIF as in Willenborg model, and η is the stress relaxation coefficient and expressed as

$$\eta = 1 - (1 - \Delta K_{th}/\Delta K)/(\gamma_c - 1) \tag{10}$$

The two adjustable material parameters are ΔK_{th} and γ_c now.

Typical simulations of overload test using the above models are shown in Fig. 2. The test was done on a CCT (Central Cracked Tension) specimen made of Al 2024-T3 with only one overload (OLR = 1.62) inserted into CA (Constant Amplitude $\Delta\sigma = 48.228$MPa, $R = 0.25$) loads. The analyses shown in Fig. 2 tell that the above models are correctly implemented in the analysis program which developed in this research as mentioned before. Furthermore, we found that this analysis program simulated the simple one-overload-only case quite well.

However, it has been shown (Chen et al, 1998) that the same set of parameters did not suit for all cases of different overload ratios, which means difficulty to choose the necessary parameters for a random loading sequence. That difficulty can be best illustrated by the simulations of multi-overload test as shown in Fig. 3.

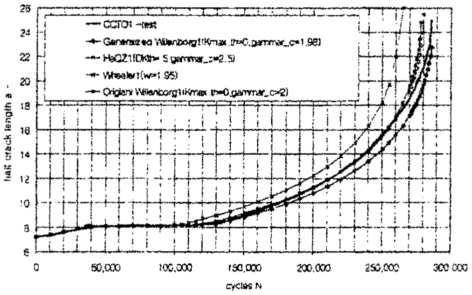


Fig. 2 Test a-N curve and simulations for single-overload test CCTO1

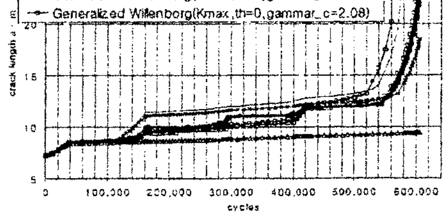


Fig. 3 Test a-N curve and Siumulations for multi-overload test CCTO3

In Fig. 3 the thick solid line is the test a-N curve for a CCT specimen with four overloads within basic CA loads ($\Delta\sigma = 48.228$MPa, R = 0.25). The four overloads are slightly different (111.118, 111.503, 109.832 and 112.533 MPa), or can be said roughly the same. Each overload was applied after the crack growth rate fully recovered from the previous overload influence as can be clearly seen from the test curve.

Predictions made by the above models will be examined in detail.

The original Willenborg model has no adjustable parameter and showed no sign of recovery after any overload, so it is not suitable to simulate this load profile.

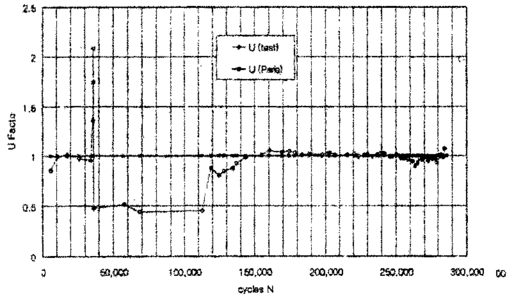
The Wheeler model with $w = 1.54$ predicted a comparable total life to test life but individual overload retardation region can not be correctly simulated. The retardation region predicted is too small for the first overload, but too big for next two overloads that CGR (Crack Growth Rate) did not recover at all. Reducing parameter w to 1.5 results in shorter retardation regions predicted for the first and the last overloads hence a shorter life than the actual test, but still the simulation showed little sign of recovery after the second and third overloads. It is impossible to simulate retardation behavior for all four overloads correctly with a single value of w. The Wheeler model would predict the same amount of retardation for each overload with the roughly same overload stresses. But the test curve showed that the retardation region becomes bigger as crack grows longer hence overload SIF becomes bigger even for the roughly same overload stress. This fact suggests that the retardation may be a function of overload SIF instead of $Cd = (r_{p_i}/\lambda)^w$.

Even with two adjustable parameters ($K_{\max,th}$, γ_c), the Generalized Willenborg model is only slightly better than the Wheeler model. Though it predicted the third retardation region better it still cannot resemble the rate recovery after the second overload.

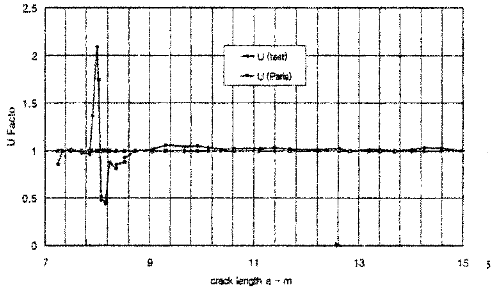
3. The Proposed New Model Using Crack Closure Concept

Elber's closure concept can be used to explain the overload retardation by assuming crack closure stress becomes higher after an overload. Here we only consider the closure caused by overload alone, the familiar Eq. (8) can be used to represent the effective SIF range.

As for Wheeler's model, from tested results the retarded da/dN after overload can be experimentally obtained, and the no retardation da/dN can be obtained by fitting test points into a Paris Equation (or some other suitable equations) without including those points affected by overload retardation. With parameters C and n (at the Paris' equation) known and the test da/dN values also known, the corresponding U factors then can be obtained as shown in the following Fig. 4. The relationships between U vs cycles, and U vs crack lengths are shown in Fig. 4(a) and Fig. 4(b), respectively.



(a) $U-N$



(b) $U-a$

Fig. 4 Curve of U factor for CCTO1

Two salients can be clearly seen in Fig. 4. Firstly for regions outside overload/retarded region the test data fall well within a very narrow band around the theoretical Paris law ($U=1.0$) line which gives much credibility of the approach. Secondly the retarded region is about the size of overload plastic zone and the tested U data in this region show gradually recovery from minimum to normal rate (U factors change from minimum after overload approaching 1.0 during recovery).

Some sort of empirical function for U can be assumed based on experiment results. Models based on simple and flexible function can be used to simulate FCG behavior under overload conditions. Inspired by the discussion in the last section on problems associated with Wheeler model we think the retardation is controlled by the current and overload SIF instead of plastic zone size. Here we propose a new model (Chen-Lee model) using a simple empirical yet very powerful expression of

$$U=(K_{\max}/K_{ap})^{m} \qquad (11)$$

where K_{ap} is the same boundary SIF as in Willenborg model, $K_{\max}$ is the maximum SIF in the current fatigue cycle.

This U factor expression has the desired features that is 1.0 outside the retarded region and it varies cycle by cycle when current SIF varis within K_{ap} dominated region. Using this new model, the predicted crack growth rate (CGR) for CCTO1 test is plotted against test data as shown in Fig. 5. This figure clearly show the suitability of the proposed model based on Eq. (11). The predicted $a-N$ curve compared with

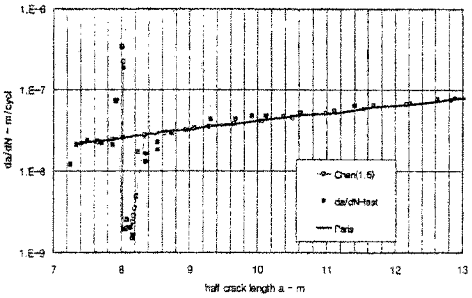


Fig. 5 da/dN comparison for CCTO1

test one is shown in Fig. 6. Comparing Fig. 6 with Fig. 2 it is seen the predicted curve using present model is the closet to the test curve. These figures show the agreement between the present model and the test results really very good.

The justification of the present model would not be complete if not checked with multi-overload test results. We are using the same test CCTO3 to illustrate the suitability of the new proposed model.

With this new model, two simulations (different m) for multi-overload test CCTO3 are made and shown in Fig. 7. It is seen clearly that the new model did correctly predict retardation region and recovery after each overload which was not possible with other models. Comparison between Fig. 7 and Fig. 3 shows clearly the improvement of the new model over the other models. In Fig. 3 no model can predict rate recovery after the second overload retardation, while in Fig. 7 the newly proposed Chen-Lee model predicts obvious crack growth after the second overload retardation region before the third overload. The improvement of the new model over the other model is essentially qualitative.

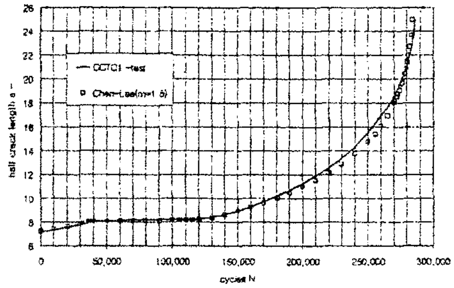


Fig. 6 Comparison of crack growth curves for CCTO1

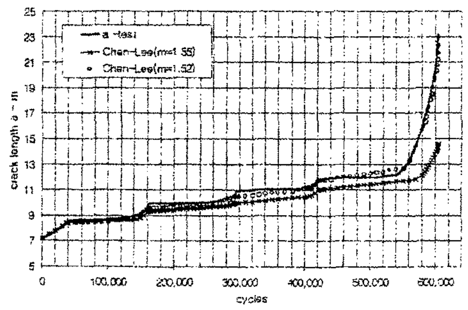


Fig. 7 Simulations by Chen-Lee model for test CCTO3

4. Simulation of Complex Load Sequence

The interaction effect includes both overload retardation and under-load acceleration phenomena. Apart from the overload retardation effect, the so-called "under-load" effect, sometimes also called "acceleration" phenomenon-much of the retardation disappears if a very low level minimum stress follows the overload-, is also of importance for a prediction model to work properly.

We have tested some CCT specimens under specially designed load profile to investigate the load interaction effects. One test case will be described here and used to examine the suitability of the above models for FCG life prediction under complex load sequences. The CCT specimen was mostly subjected to CA fatigue cycling with some overload and over/under loads applied at various points. The CA cyclic parameters are as following, maximum stress = 80 MPa, minimum stress = 40 MPa, fatigued at frequency of 7Hz. Three overloads of 130 MPa stress were applied after 50,000 CA cycles with a 500 CA cycles in between. Other overloads of 130 MPa followed immediately by under-load of 20 MPa were applied at points after 190,000 cycles and 250,000 cycles (after completely recovered from the previous load interaction influence).

In Fig. 8, the joint line between the dark square points is the test curve. Also shown in the Figure

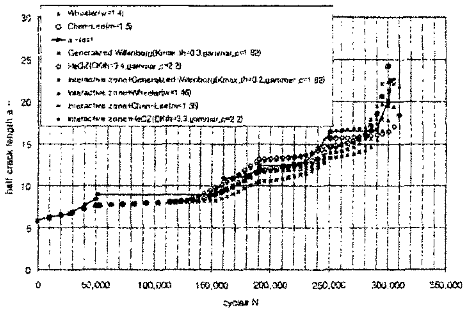


Fig. 8 Predictions for a complex load sequences compared with test

are some predictions made by the various models discussed above. The original Willenborg model has no adjustable parameter so it can not predict even correct tendency not to mention the complete a-N curve for this load sequence. The original Willenborg model predicted a much smaller retardation region than the actual test and reached critical crack length even before other over/under loads were applied. The Wheeler model (triangles) predicted comparable retardation region for first group of three overloads, but not so good for the other two over/under load combinations, especially in the third retardation region the predicted curve is too far away from the test curve. The Generalized Willenborg model prediction is more or less the same the Wheeler model-on one hand the predicted total life is more closer to test life than the Wheeler model, on the other hand the middle part of the predicted curve differs from the test curve more than the than the Wheeler model. The HeQZ model (empty diamonds) predicted less retardation for the first region but still a longer total life compared with test curve. Among these curves the $a-N$ curve predicted by the present model (line jointed by dark diamonds) is most close to the test curve generally and in total life. It shows the best prediction is given by the newly proposed Chen-Lee model even for the complex load sequence examined here.

5. Conclusion

We have examined some existing retardation models for FCG life prediction in light of test results. It is found that though those models worked well for single overload case they could not correctly simulate multi-overload test behavior. A new model (Chen-Lee model) considering "extra" crack closure was proposed. The model used a simple yet very powerful expression to calculate U factor cycle by cycle. The calculated crack growth rate da/dN and the predicted a-N behavior compared well with test points as shown in Fig. 5 and Fig. 6. The superiority of the new model is further demonstrated through correctly simulating multiple overload test as shown in Fig. 7. Finally the new model is successfully applied to complex load sequences as shown in Fig. 8. It shows the best predictions are given by the newly proposed Chen-Lee model for both the multiple overload test and the complex load sequence examined here.

Acknowledgement

This study has been supported by 1997 Brain Pool Program of the Korean Federation of Science and Technology Societies, RRC (KOSEF) at Inha University, and SAFE (KOSEF) at Sung Kyunkwan University.

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